Slide 1

One problem and one solution

The problem: “time-varying propagation speed in waveguides”

$$s(t + d^+(t)) = e(t) \tag{1}$$

A solution: “fractional delay filter”

$$s(t) = e(t - d^-(t)) \tag{2}$$

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Forward and backward delay I

- We define two time scales τ^+ and τ^- in such a way that:

$$\left\{ \begin{array}{ll} \tau^+ : & t \mapsto t + d^+(t) \\ \tau^- : & t \mapsto t - d^-(t) \end{array} \right. \text{ or, said differently } \left\{ \begin{array}{l} \tau^+ = \mathrm{Id} + d^+ \\ \tau^- = \mathrm{Id} - d^- \end{array} \right. \tag{3}$$

- Equations (1) and (2) are recasted as:

$$\left\{ \begin{array}{rcl} s \circ \tau^+ & = & e \\ s & = & e \circ \tau^- \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \tau^+ \circ \tau^- = \mathrm{Id} \\ \tau^- \circ \tau^+ = \mathrm{Id} \end{array} \right.$$

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Forward and backward delay II

- $\boxed{\tau^+ \circ \tau^-} = \mathrm{Id} \;\Rightarrow\; \tau^- + d^+ \circ \tau^- = \mathrm{Id} \;\Rightarrow\; \boxed{d^+ \circ \tau^- = d^-}$
- $\boxed{\tau^- \circ \tau^+} = \mathrm{Id} \;\Rightarrow\; \tau^+ - d^- \circ \tau^+ = \mathrm{Id} \;\Rightarrow\; \boxed{d^- \circ \tau^+ = d^+}$
- Finally we have: $\forall t,\ \begin{cases} d^+(t - d^-(t)) &= d^-(t) \\ d^-(t + d^+(t)) &= d^+(t) \end{cases}$

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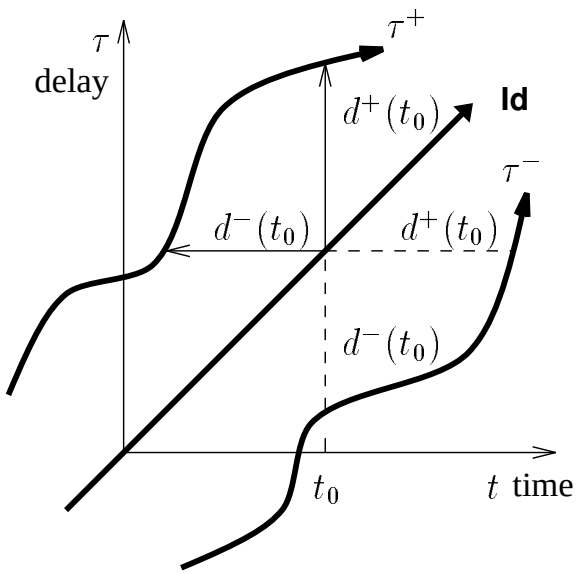
Graphical Interpretation



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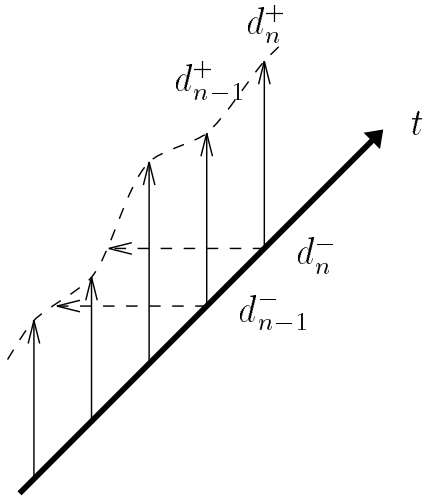
Discretisation I

Graphical interpretation:



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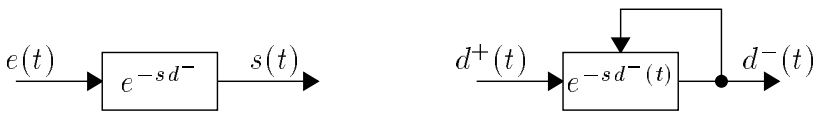
Discretisation II

Continuous-time system:

$$\begin{aligned} d^-(t) &= d^+(t - d^-(t)) && d^+ \text{ to } d^- \text{ convertion} \\ s(t) &= e(t - d^-(t)) && \text{fractional delay equation} \end{aligned} \tag{4}$$



$s(t) = e(t - d^-)$ $\qquad$ $d^-(t) = d^+(t - d^-(t))$

Figure 1: Formal interpretation

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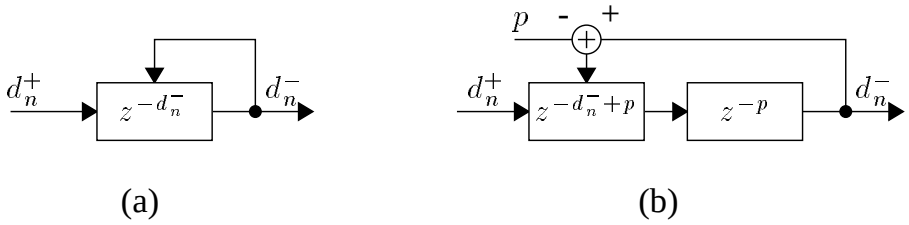
Discretisation III

Discrete-time system:



(a) (b)

Figure 2: Ideal discretisation

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Complete discrete-time model

Let us suppose an one-to-one relation between the input pressure e and the *forward* delay d^+: $\boxed{d^+ = f \circ e}$

Then the output pressure may be computed as:

$$s = e \circ \tau^- = f^{-1} \circ f \circ e \circ \tau^- = f^{-1} \circ d^+ \circ \tau^- = f^{-1} \circ d^- \qquad (5)$$

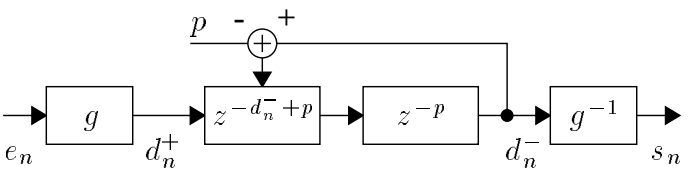

Figure 3: Complete model of an one-way time-varying propagation speed waveguide

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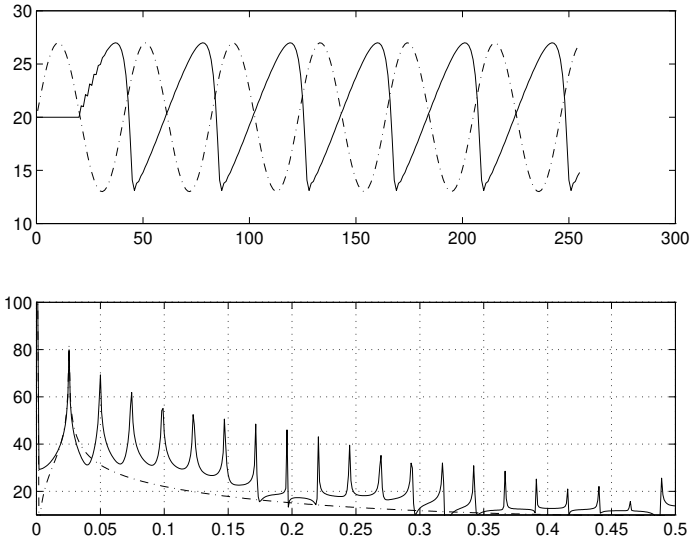
Simulation



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Summary

- a nonlinear filter which converts a *forward* delay into a *backward* delay,
- in the case of a one-to-one relation between the input pressure and the *forward* delay, only *one* fractional delay filter is needed for simulating the nonlinear propagation,
- a Borned Input / Borned Output nonlinear filter.