Uncertainty in Distinguishing between Structure and Ornamentation in Music

Dalia Cohen Shlomo Dubnov Naftali Wagner
Musicology Department, Institute for Computer Science
Hebrew University, Jerusalem 91904, Israel

Abstract *The paper focuses on the uncertainty inherent in the ornamentation/structure pair and on the role of uncertainty in characterizing style. We suggest that the ornamentation/structure pair be viewed as "complementary opposites" that fulfill three conditions: contradiction, support and identification. In computer simulations we perform a detailed harmonic analysis of a Prelude by J.S.Bach and suggest applicability of this for characterizing style. A full version of the paper is in* **ftp.cs.huji.ac.il:/pub/il-cmf/Uncertainty-Bach.ps**

1 Introduction

Ambiguity in music is increasingly preoccupying researchers in both retrospective and real-time analysis. In the first case, the ambiguity is caused by the disparity between the actual musical states and the dictates of analytical tools that rely on a theoretical model, and by the lack of a standard theory of ambiguity [1],[2],[14] ,[9] ,[16]. In the second case the ambiguity concerns the ability (or inability) to predict the continuation of the musical progression, and this is related to the system of expectations [12], [6],[7],[8],[13],[4], etc.. Here we assume not only that the ambiguity/definability pair plays an extremely important role in shaping the artistic experience, but that it can be viewed as a "hyperparameter" that contributes to the structure of the piece and plays a part in characterizing the style. (Let us remember that two opposing ideals, "ethos" and "pathos" [15] , are characterized in part by the ambiguity/definability pair .) This parameter characterizes the musical raw material and specific pieces both by how common the ambiguity is and how it is manifested and by the dynamics of variability throughout the piece. Ornamentation, by its very nature, indicates a hierarchical distinction between the ornament and the ornamented. In musical organization, the ornamented item represents a structural item, but in the hierarchical organization of events any ornamented item on one level may, in a sense, be ornamentation for a structural event on a higher level. Because any or all of the relevant variables (difference/similarity, salience/nonsalience and stability/instability) may be marked by ambiguity, it is all the more likely that the function - ornamental or structural - defined by those variables will be ambiguous. Whether the function is structural or ornamental may be determined with reference to both learned schemes (which are not necessarily arbitrary) and "natural schemes." The function - structural or ornamental - in the learned schemes, which are determined in the cognitive stage, is expressed by whether or not an item is part of the scheme (e.g., scale, rhythmic pattern, chord, harmonic phrase). Each scheme may have a different amount of ambiguity, and the likelihood of ambiguity increases the farther we move from an optimal size for the segment, that is, toward the shortest or longest segment. The natural schemes, some of which involve psychoacoustic constraints and others cognitive constraints, concern diverse variables, such as salience/non-salience, repetition of various sorts, and ability to predict the continuation of the progression. These variables may have different degrees of ambiguity. Thus, ambiguity with respect to ornament/ornamented may stem from ambiguity in the learned schemes, from non-concurrence between schemes on various levels, and from non-concurrence between learned and natural schemes. As a result, we can define the conditions for the emergence of structural or ornamental events as follows:

- Conditions for a structural event: being part of the scheme, stability, distinguishability
- Conditions for an ornamental event: not being part of the scheme, instability, indistinguishability, proximity to the structural event in terms of the various parameters.

As noted, these conditions refer both to learned and to natural schemes, but the numerous possibilities of non-concurrence may produce different degrees of ambiguity in the structure-ornament relationship. Moreover, the fact that the ornament-ornamented relationship may appear on various levels produces various relationships, some of which contradict each other: (1) intensifying the ornamentation may **weaken** the supremacy of the ornamented item, so that extensive ornamentation on the immediate level diminishes the possibility that broader levels will be formed; (2) the ornamentation can also **accentuate** the ornamented, causing the immediate emergence of an additional level; (3) multiple levels cause an **exchange of roles** between some of the ornamentation and some of the ornamented items. The ornament/ornamented relationship on the various levels can be regarded as a fractal structure: the same principles can be found in any order of magnitude.

Another relevant factor concerns the ideals of various cultures, which differ, inter alia, in the degree of ambiguity preferred and the levels on which it appears. This has to do with the role of music in cultural activity in general, as well as with the relationship between cultural activity and its natural environment. For example, the ideal in the Western musical culture calls for composition of a complex, long-term piece that forms a complete, independent unit. This requires that the levels be separated fairly clearly. This separation requires avoidance of excessive ornamentation on the immediate level, which would otherwise prevent the formation of broader levels in accordance with Von Forster's theory [10]. In other cultures, where the ideal calls for the music to interrelate with its environment, such that the piece is not a separate, independent unit, ornamental multiplicity occurs on the immediate level. This multiplicity answers the need to perpetuate the moment and the need to avoid a tight superstructure that cuts the piece off from its environment. The Western ideal, which, from a broad perspective, attempts to reduce ambiguity, is divided into sub-ideals (according to era, geographical area, composer, etc.), which can also be characterized by ambiguity.

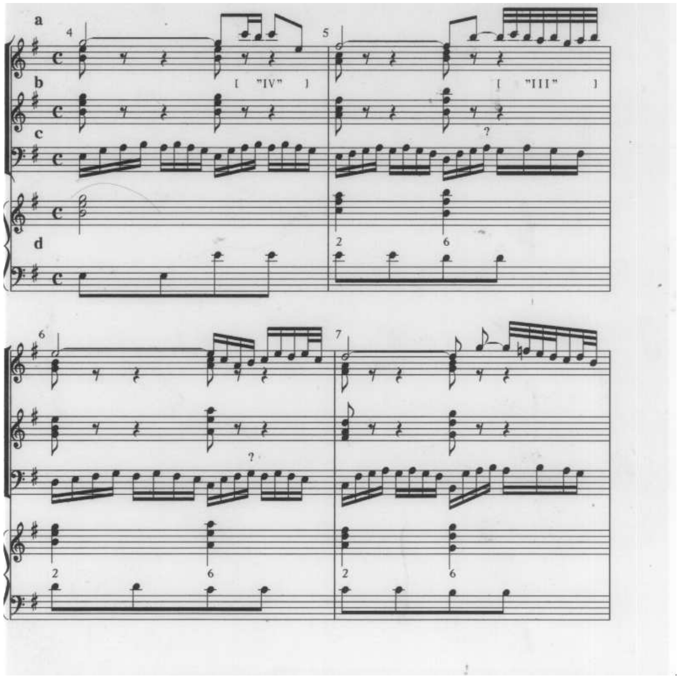

Figure 1: The three versions of the Prelude (bars 4-7): a) BWV 855/1 (WTC-1), Right Hand b) BEV 855a/1, Right Hand c) BWV 855/a & 855a/1, Left Hand d) Bach, Thorough Bass Pattern

2 Bach's Prelude

To illustrate, we have selected a piece by Bach - the Prelude in E Minor, from the first part of the Well-Tempered Clavier (WTC) - in which ambiguity, in its various manifestations, is notable; this ambiguity is consistent with Bach's stylistic ideal, which calls for an extremely complex integration of ornamentation and structurality. This prelude is based on a simpler, shorter prelude (BWV-855a) that Bach composed for didactic purposes and which is itself based on a harmonic-contrapuntal sequential scheme (d in the example) that can be found in his theoretical-didactic writings [5]. The three versions can be seen as three stages in the creative process; each stage is, in a sense, ornamentation for the stage below it: the brief prelude expands the didactic scheme and adds a lower decorative layer played by the left hand and composed of a melodic figure of notes of equal durations, and its contour recurs throughout the piece (c in figure 1).

This figure functions as a broken chord with foreign notes added. As early as this stage we are faced with the problem of which note belongs to the chord (the structural part) and which is foreign (ornamental). However, Bach consistently takes advantage of fine distinctions between the figures for the left hand to contribute to the overall structure. The WTC version adds an upper, flexible, irregular, "improvised" layer (a in the example). The three strata in the prelude (a, b, c) represent three degrees of ambiguity. In the middle one the affinity for the original didactic scheme (d) is noticeable; therefore it is the clearest and the upper one is the least clear. Furthermore, when one looks at the relationship between the layers, the uppermost layer is the main source of the ambiguity in the prelude, as it disrupts the sequence and interacts in various ways with the other strata in the texture to cause the momentary formation of chords that deviate from the didactic scheme. Similarly, some ornaments in the upper layer (a) recur in various chords, creating competition between the ornaments and the chords for the listener's attention. The prelude in WTC also expands on the horizontal axis, and it includes a sort of broad coda that almost doubles the length of the prelude. This supplement forms, in a sense, an improvised, virtuoso repetition of the previous part, but strictly preserves sections from the bass layer. In this part the ambiguity is intensified by giving the lower layer various harmonic interpretations.

Below we provide a formal analysis of one aspect of ambiguity: the degree of belonging to the chord scheme.

3 The Problem of Chord Estimation

Our formal treatmet of the uncertainity aspects in music will be limited to the problem of chord estimation. Rules of harmony, which are largely treated in musical textbooks, are in a sense abstractions of principles of pitch organization. Discovering the harmonic structure in a tonal music piece, being one of the basic tasks of any musical analysis, might be sometimes a non-trivial task when dealing with polyphonic music of contrapunctal texture (many independently moving voices). The harmonic structure of the Prelude (according to our human analysis) is demonstrated in Fig.2. In the following we shell take some assumptions

1 :	$2e$	$2f\sharp 0_2$	$1d\sharp o_7 B_{\substack{5\\6}}$	$2e$
5 :	$1f\sharp 0_2\ 1b_6$	$1e_2\ 1a_6$	$1D_2\ 1G_6$	$1C_2\ 1f\sharp o_6$
9 :	$2G$	$1A_2\ 1D\sharp_6$	$2e$	$1F_2\ 1bo_6$
13 :	$2C$	$1D_2\ 1g\sharp 0_6$	$1a1a_2$	$1d\sharp_6 o1a_{\substack{4\\6}}$
17 :	$1D\sharp 1B_{\substack{5\\6}}$	$1E_2\ 1a\sharp o$	$1f\sharp 0_{\substack{3\\4}}\ 1e_{\substack{4\\6}}$	$1a\sharp o_{\substack{5\\6}}\ 1B$
21 :	$2e$			

Figure 2: A human analysis of the Prelude (first 21 bars).

concerning the nature of pitch organization :

(1) Our basic elements are notes and collections of notes over a short segment in time , which we call "windows".

(2) A single note "belongs" to several chords and the extent of this belonging will serve as the basis for chord estimation.

(3) The contributions of the notes in a window define a *Harmony vector* which gives the merit of belonging of the window contents to a set of chords. The estimated current chord is chosen to be the one at the vector maxima.

(4) We assume that this belonging measure is independent of the melodic, rythmic or other musical processes. We also assume that no interaction occurs between the adjacent notes or voices so that the evidence can be independently accumulated by summing the contributions of each note in the window.

(5) The chords are estimated completely on local window basis. No context dependencies are taken into account, such as schemes of harmonic progressions.

(6) "Natural" candidates for windows are binary subdivisions of the musical meter, as indicated at the left end of the musical staff. The prevailing chord "governs" the pitch organization for a limited time interval and our task is to find the correct window.

Let us define by $P_x(C)$ the relative contribution of a note x to the evidence of appearance of a chord C. This could be seen in a fuzzy interpretation as a grade of membership of a note with respect to the various chords. In probability terms one could say that this is the probability to have a chord C given an observation of note x. The probability for a chord in a window W is given by $P_W(C) = \sum_{x \in W} P_x(C) P_W(x) = \sum_x P_x(C) N_x / N$ $P_W(x) = N_x/N$ is the empirical probability with N_x the count of appearances of x in the window W and $N = \sum_x N_x$. In order to estimate the chord structure in the piece we build a series of weighted histograms as estimates of the momentary chord probability for each window. The histograms are calculated by summing the contribution of notes over window segments from the piece. The temporal location of the window determines the time coordinate and the window size is described by a scale index. We chose to take non-overlapping windows that musically correspond to the natural bar divisions. The scalings (zoom out or in) are multiples or subdivisions of a window by factors of 2. This series of windows give us a multiscale, time variant representation of of the harmonic contents present in the musical work. We use a pair on indices $W = (t, \alpha)$ to denote a window at discrete time t and scaling α. As mentioned above, each note could be a root, third or a fifth of a chord (and also a seventh in a broader harmonic language) when it appears as a structural element, or any other note when it is a passing, sustained or any other embellishing tone. In the last case the main function of the note is melodic and with respect to the harmonic analysis it is viewed as an element that "blurs" the structure.

4 Results

Our chord alphabet consists of major and minor chords (triads and a dominant seventh chord), diminished chords (triads and septachords) indicated by suffix o and half diminished sepetachords indicated by 0, a total of 48 chord names. The typical harmonic rhythm in the Prelude is two quarters (half bar). The first simulatinos were performed using a constant length window. A window of two quarters duration gave the best results which corresponded nicely to human judgments. At the second step we deviced a method for merging neighbouring windows of high similarity into a higher level window. Before turning to this few words about the entropy of the harmony vector are in place.

4.1 Harmony selection and its Entropy

The results of chord estimation depend strongly on the choice of the window W. A window that corresponds to "correct" harmonic rhythm gives good results, while shorter windows give unnecessary level of detail and large windows average out the real events. Our multiscale analysis actually presents a complete binary tree of harmony vectors with a node $W = ((t+1)/2^{\alpha+1}, \alpha+1)$ branching into $W = (t/2^\alpha, \alpha)$ and $W = (t/2^\alpha, \alpha)$, $\alpha = 0, .., \textit{highest allowed level}$. Our task now is to "trim" the unnecessary branches so as to reach at each time point a window at the correct level.

In the following we present a merging algorithm that performs such a trimming by clustering neighboring $P_W(C)$'s of high similarity. In general, given a certain error threshold, we start passing on the tree left to right from the lowest level (in our case a reasonable minimal level corresponds to a quarter note long segment). During the pass we check the adjacent branches for their relative distance. If the distance is lower then the allowed error, we "merge" by ascending to a higher level and repeat the process recursively.

Before proceeding to the description of the algorithm it is in place to say a few words about measuring distances between harmony vectors.

The distance measure of our choice was the *relative entropy*, known also as the Kullback-Liebler (KL) [11] divergence $D(P_W||P_{W'}) = \sum_C \ln P_W(C)\frac{P_W(C)}{P_{W'(C)}}$ which measures a distortion between probability distributions. The merging algorithm uses this distance function to decide when to perform the merging. The algorithm is summarized in the following box:

- Initialize $W = (t = 1, \alpha = 0)$
- **Merge(W) :**
  1. Pick the harmony vector at W.
  2. If W is a left branch (odd t) **OR** no predecessorexists on the same level α, **Merge($W = (t+1, 0)$)**.
  3. If $D(P_{((t-1)/2^\alpha,\alpha)}||P_{(t/2^\alpha,\alpha)}) < Err$, delete the two branches; **Merge($W = (t/2^{\alpha+1}, \alpha+1)$)**.
  4. Else **Merge($W = (t+1, 0)$)**.

Figure 3: The algorithm for merging the harmony tree.

The results of applying the merge algorithm to the Prelude are demonstrated below (Fig. 4). [1]

1 :	$2e$	$2f\sharp 0$	$2B^+$	$2e(3e\ 1a)$
5 :	$1f\sharp 0\ 1b$	$1e\ 1a$	$1D\ 1G$	$1C\ 1D^*$
9 :	$2G(3G\ 1a)$	$1A\ 1B^*$	$2e$	$1F\ 1G^*$
13 :	$2C(3C\ 1d)$	$1D\ 1E^*$	$4a^+$	
17 :	$2f\sharp o^{**}$	$1E\ 1a\sharp o$	$1f\sharp 0\ 1e$	$1c\sharp o^{**}\ 1B$
21 :	$2E^+$	$1a\ 1E$	$4a^+$	
25 :	$2E$	$2a$	$1c\sharp o\ 1B^*$	$1e\ 1d$
29 :	$1a^?\ 1b^?$	$1a\ 1C^?$	$2a$	$1B^*\ 1e$
33 :	$2a^?$	$1F\sharp^*\ 1B$	$1G\ 1a^?$	$1e\ 1B$
37 :	$1f\sharp^?\ 1B$	$2e^?$	$1f\sharp^?\ 1B$	$1C\ 1a$
41 :	$1B\ 1E$			

Figure 4: The results of merging the harmony tree.

[1] We use brackets and the small marks on the upper right side of some of the chords to designate the following important cases:
() We have chosen to demonstrate three typical cases of ornament/structural ambiguity (bars 4, 9, 13). The results in brackets were achieved by applying a one quarter note window. For instance, in bar 4 we have an a chord on the last quarter, which results from the soprano voice line that "belongs" to the a harmony if analyzed isolatedly. (The numbers in the brackets are quarter note counts.)
$*$ In case of diminished triad, the program chose a chord a third below, which contains the diminished as a septachord and they both serve the same harmonic function.
$**$ In case of diminished septachord the program chose a chord a third above, which is enharmonically equivalent (contains exactly the same notes) and musically simply is an incorrect naming of a note.
$+$ Merging of ambiguous chords exceeded the desired level - for instance in case of a chord progression on top of a pedal tone, the musicologist would prefer to observe the chord progression rather then merging it into one harmony.
? Ambiguity in the text. The results were erroneous and please note that this occurs only in the second part of the Prelude.

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